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$+(2ri)^2+(2rj)^2+(2rk)^2+(2rl)^2+(2rm)^2+(2rn)^2+(2rp)^2+(2rq)^2\ldots.(5)$. Take $i=1)$, $j=2$, $k=3$, $l=4$, $m=5$, $n=6$, $p=7$, $q=8$, $r=9$; then, after dividing the numbers by 3, we have $6^2+12^2+18^2+24^2+30^2+36^2+41^2+42^2+48^2=95^2$.

An infinite number of sets of nine square numbers whose sum is a square may be found from (5).

27. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Decompose into the sum of two squares the product $5\times13\times61$.

I. Solution by DAVID E. SMITH, Ph. D., Professor of Mathematics, State Normal School, Ypsilanti, Michigan.

Taking the usual formula

$$(m^2+n^2)(x^2+y^2)=(mx\pm ny)^2+(my\mp nx)^2,$$ and noticing that $5.13.61=65.61$, the problem reduces to decomposing 65 and 61 into two squares, which give

$$65=64+1=49+16,$$
$$61=36+25.$$
$$\therefore\ 65.61=53^2+34^2=43^2+46^2=62^2+11^2=22^2+59^2.$$

The products 1.3965, 5.793, and 13.305 give no different results.

II. Solution by the PROPOSER.

$$(m^2+n^2)(p^2+q^2)=(mp\pm nq)^2+(mq\mp np)^2,=A^2+B^2.$$
$$(m^2+n^2)(p^2+q^2)(r^2+s^2)=(A^2+B^2)(r^2+s^2),=(Ar\pm Bs)^2+(As\mp Br)^2.$$

Let $m=2$, $n=1$, $p=3$, $q=2$, $r=5$, $s=6$.

Then $A^2+B^2=(6\pm2)^2+(4\mp3)^2,=8^2+1^2,=4^2+7^2$.

$$(2^2+1^2)(3^2+2^2)(5^2+6^2)=5\times13\times61$$
$$=(40\pm6)^2+(48\pm5)^2,=(20\pm42)^2+(24\mp35)^2.$$
$$=46^2+43^2,=34^2+53^2,=62^2+11^2,=22^2+59^2.$$

III. Solution by COOPER D. SCHMITT, Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

I present a general solution of this problem, true for any number found according to the same law.

$5=1^2+2^2$, $13=(1+2)^2+(1\times2)^2,=3^2+2^2$. $61=(3+2)^2+(3\times2)^2=5^2+6^2$.

Then in general we have $(x^2+y^2)[(x+y)^2+(xy)^2][(x+y+xy)^2+(xy)^2(x+y)^2]$ and this is to be shown to consist of two squares.

Using determinants we have as follows:

$$\begin{vmatrix} x & -y \\ y & x \end{vmatrix} \times \begin{vmatrix} x+y & -xy \\ xy & x+y \end{vmatrix} \times \begin{vmatrix} x+y+xy & -x^2y-xy^2 \\ x^2y+xy^2 & x+y+xy \end{vmatrix}$$

$$=\begin{vmatrix} x^2+xy+xy^2, & x^2y-xy-y^2 \\ xy+y^2-x^2y, & xy^2+x^2+xy \end{vmatrix} \times \begin{vmatrix} x+y+xy & -x^2y-xy^2 \\ x^2y+xy^2 & x+y+xy \end{vmatrix}$$

$$\left|\;=\begin{matrix}(x^2+xy+xy^2)(x+y+xy)+(x^2y-xy-y^2)(-x^2y-xy^2), \\ (xy+y^2-x^2y)(x+y+xy)+(xy^2+x^2+xy)(-x^2y-xy^2),\end{matrix}\right.$$

$$\left.\begin{matrix}(x^2+xy+xy^2)(x^2y+xy^2)+(x^2y-xy-y^2)(x+y+xy) \\ (xy+y^2-x^2y)(x^2y+xy^2)+(xy^2+x^2+xy)(x+y+xy)\end{matrix}\right|$$

$$=(x^3+2x^2y+2x^2y^2+xy^2+xy^3+x^3y+3x^2y^3+xy^4+x^3y^2-x^4y^2-x^3y^3)^2$$

$$+(x^4y+3x^3y^2+x^3y^3+x^2y^3+x^2y^4+x^3y-x^2y-2xy^2-y^3-xy^3)^2.$$

If $x=1$, $y=2$, as in given problem, I get $5\times13\times61=59^2+22^2$.

If $x=2$, $y=3$, I find $13\times61\times2021=398^2+807^2$ and so on indefinetly.

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AVERAGE AND PROBABILITY.

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

14. Proposed by CHARLES E. MYERS, Canton, Ohio.

$\frac{1}{3}$ of all the melons in a patch are not ripe, and $\frac{1}{4}$ of all the melons in the same patch are rotten. the remainder being good. If a man enters the patch on a dark night and takes a melon at random, what is the probability that he will get a good one?

I. Comment by JOHN DOLMAN, Jr., Counsellor-at-Law, Philadelphia, Pennsylvania.

The published solution of Probability problem No. 14 is erroneous. The simplest correct solution is as follows:

If the melon selected is *not unripe* and *not rotten*, it will be good. The chance that it is unripe is $\frac{1}{3}$, therefore the chance that it is not unripe is $\frac{2}{3}$; by similar reasoning, the chance that it is not rotten is $\frac{3}{4}$. Therefore the chance that it is not unripe and not rotten, is $\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{2}$, which is the chance required.

It is not very difficult to point out the error in the published solution. While it is true there cannot be more than $8n$ nor less than $5n$ good melons, it does not follow that $\frac{1}{2}(8n+5n)$ is the average or most probable number, unless it be predicated that all values between $5n$ and $8n$ are equally likely, which is not the case.

Dropping the n's, suppose there are 8 ripe and 4 unripe melons, the three rotten ones may be selected from these 12 in 220 different ways each of equal probability. Now a very simple application of the principles of choice will show that of these ways

4 would leave 8 good melons
48 would leave 7 good melons
112 would leave 6 good melons
56 would leave 5 good melons